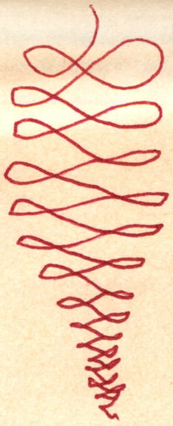


HS	C	W	0/3	1/3	2/3
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
0 - $\frac{2}{3}$	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{2}{3}$
$\frac{1}{3}$ - 1	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$
$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

1	1	1
0	1	2
3	2	1
2	2	2

$(1-\lambda_i$

$= \lambda$

$\zeta_1\lambda_1 + \cdots\cdots + \xi_n\lambda_n$

$p_1\lambda_i + p_2\lambda_2 + \cdots\, p_n\lambda_n$

$\lambda_i \cdots\cdots$

			$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	
III	III = IV	IV = III				
IV		$\overset{III}{I}$ = III	1	0	0	$\frac{1}{6}$
II	II = I		0	1	0	$\frac{1}{6}$
I			0	0	1	$\frac{1}{6}$
			1	0	1	$\frac{1}{3}$
			0	1	1	$\frac{1}{3}$
			1	1	0	$\frac{1}{3}$

207

20
15
20

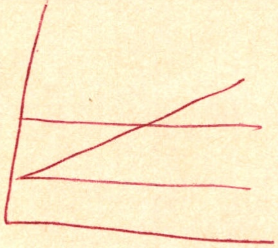
262

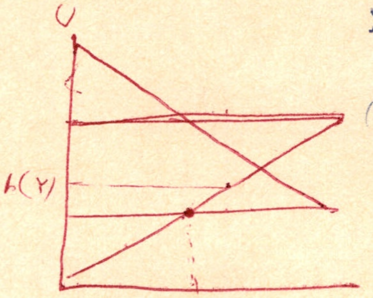
$\frac{1}{3}\cdot\frac{1}{3}\cdot 1 + \frac{2}{3}\cdot\frac{2}{3}\cdot 1 = \frac{5}{9}$

$\frac{1}{3}\cdot\frac{2}{3}\cdot 1 + \frac{2}{3}\cdot\frac{1}{3} = \frac{4}{9}$

Value of an action = expected value of the Type I probs of winning with that action.

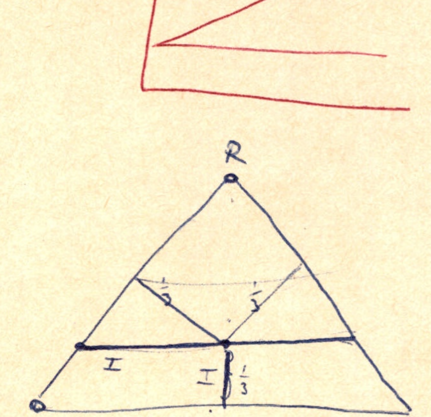
~~Expected~~





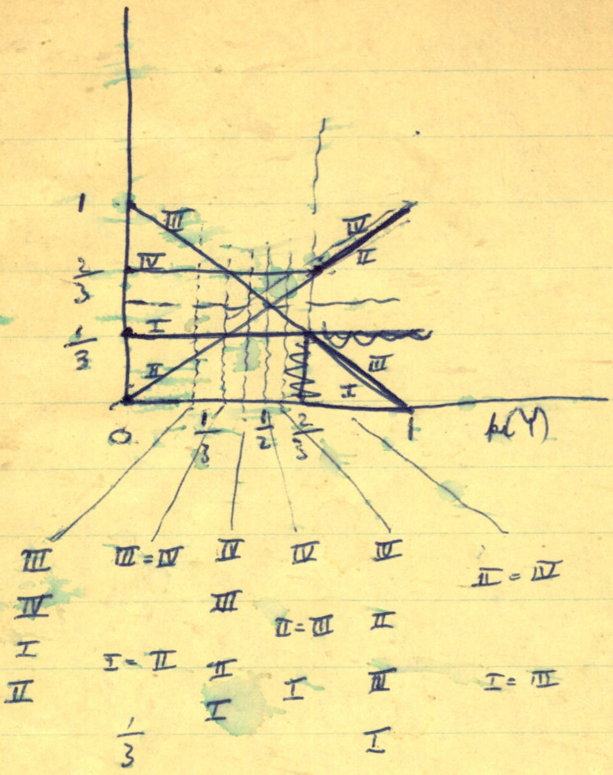
$\zeta_1(1-\lambda_1) + \cdots\cdots + \zeta_n(1-\lambda_n)$

$(\zeta_1 + \cdots\ \zeta = 1 - \xi -$

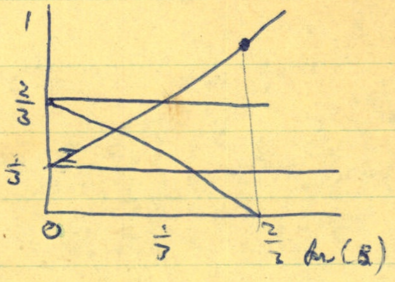


P_0 (Red)

P_0 (yellow) = $\sum p_i$ prop$_i$(Y)



	Y	
1	0	0
0	1	0
1	0	1
0	1	1

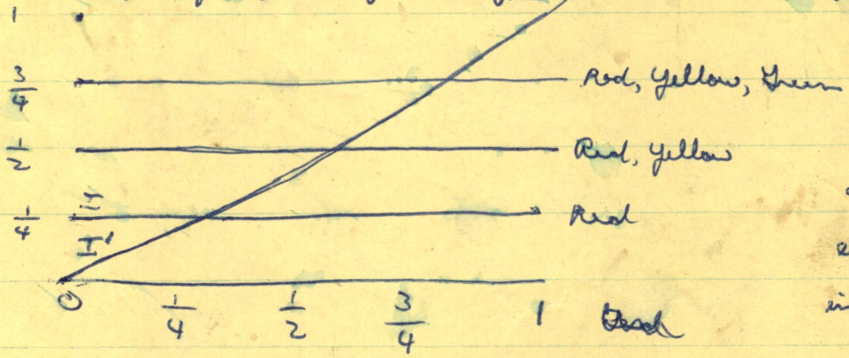


Imagine these as definite payoffs: vN-M utilities, instead of prospects.

Or: imagine them as money payoffs. (Then you wouldn't have them = vN-M)

Indep Assump — (Smith) — probability money ?

pr (1) = vN-M utility of strategy with 0,1 payoffs by P2; whether or not probs are specified (though in a game, it wouldn't be; don't assume in game that you do what is most likely to win prize; [but we could assume you max $P_o(1)$, even though Y^o doesn't include all possibilities.



Bias Toward Axiom II

Play with cards, one of which is worth $1000 (number chosen randomly).

Nixon: foreign policy debate (1st or 4th)
odds on 8 bill
McNamara: AW, 2 April

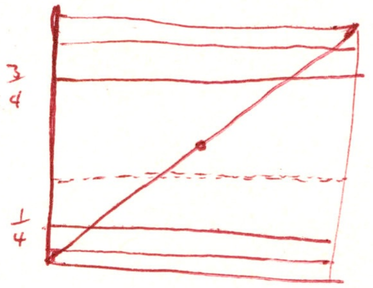
Chap I: Good on vagueness.

Chap V

In example: don't insist that you must believe (or feel) this or that;
a) think up your own circumstances, combining precise + vague probs for payoffs
b) real situations; hard to model for everyone.

Differences in recognition of difference. (ambiguity).

Chap VII ~~Rank A, B, C; safety factor~~
~~vagueness of higher ranks~~ 0, 1, 2 yellow case.
variance of utilities, or of prob. dist. (Allais); violation of utility axioms (3:4:6)
Do I believe, for definite probs? Maybe not, except his special case.



Red, $\overline{Red}$

$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Must be indifferent

~~Prob~~

Ball may be R, Y, B, G   Suppose someone peeks, says "It's not Yellow"; will prob of Red really go up? So Red ∪ Black in vague urn ⇒ 3R (out of 5) in definite urn?

1R, 3$\overline{R}$

3R, 1$\overline{R}$

Prob of win/lose biased toward $\frac{1}{2}$; i.e. toward Hurwicz condition.

(Chipman: bias toward 1/n for ~~any~~ but on any given, vague event. Yes: "best guess" effect.
But that doesn't explain preference for betting on definite event with prob = $\frac{1}{n}$; must explain by bias toward prob of losing $\boxed{>}$ $\frac{n-1}{n}$, e.g. $> \frac{1}{2}$; i.e. prob. of winning $\boxed{<}$ $\frac{1}{n}$ for every vague event (all n of them); or prob $\boxed{>}$ $\frac{1}{n}$ when betting against the event.

1

0	0	$\frac{1}{3}$
$\frac{1}{3}$	0	0

0	0	$\frac{1}{3}$
$\frac{1}{3}$	0	0

		Evaluation				
$\min_x$	$\max_x$	$\alpha=0$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
0	$\frac{2}{3}$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

1	0	0	$\frac{1}{[illegible]}$		$\frac{1}{3}$	$\frac{1}{3}$
0	1	0	$\frac{1}{6}$		$\frac{1}{6}$	$\frac{1}{3}$
0	0	1	$\frac{1}{6}$		$\frac{1}{6}$	$\frac{1}{3}$
1	1	0	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
0	1	1	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
1	0	1	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$\frac{1}{6}, \frac{5}{6}$

$\frac{1}{6}, \frac{5}{6}$

$pr\,A + pr\,\bar{A} \leq pr(A \cup \bar{A})$

$pr(R) + pr(Y) \leq pr(R \cup Y)$

$\frac{1}{3}$

R Y B

R	$\bar{R}$	Y	$\bar{Y}$	B	$\bar{B}$
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{1}{6}$	$\frac{4}{6}$
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{5}{6}$

$pr(A \cup B)$ [illegible] $pr\,A + pr(B)$

attention; not only is it the key element in the "bit of logic" that implies that you should prefer Option A to Option B if you prefer I to II and IV to III, but it would in itself rule out the latter pattern of preferences.[1] Moreover, ~~both~~ in the ~~axiomatizations of probability and utility~~ axiom systems presented by Chernoff, Luce and Raiffa, and Milnor, this axiom or a direct consequence of it plays the logical role of Postulate 2 in Savage's system; in each case it is the only desideratum violated by the minimax criterion, which it thereby rules out (along with our modification, the "restricted Bayes/minimax criterion").

---

[1] Luce and Raiffa's discussion of Axiom 8 suggests that violation of Axiom 8 is far from being the felony ~~that~~ suggested by loose mention of "strict dominance": which would invoke Axioms 4 or 5, or Savage's Postulate 3! Despite the authors' own "taste" for Axiom 8, they devote two pages (more space than any of their other 11 axioms receive) to stating, illustrating and responding to three common objections to it (I would subscribe to all three). Later, when they conclude that their Axioms 1 through 11 are mutually incompatible, they comment: "Something will have to be deleted, and one possible candidate is Rubin's axiom 8." All this, with the estimate that Axiom 8 is the "weakest link in the argument" leading to the anticonvexity property (violated by our hypothetical subjects), is in interesting contrast to Raiffa's assertion above: "I cannot see how anyone could refute the logic leading to the conclusion that given your initial choices, you should prefer option A to option B. This bit of logic is certainly not the weak link in the argument" (*op. cit.*, p. 694).

As a matter of fact, surely few readers of Games and Decisions would ~~response~~ hesitate for a moment to respond to Raiffa's expostulation, "Something must give!" by nominating Rubin's axiom.

1 -1 2 1.5 -1.5

-2 2 2 2 -2 3 -3

-4 4 0 0 1 0 |4 0|

200 0 -2 2 |4 0|

5

2 0 0 0 0 2

0 2 0 0 0 2 - 2 0 2 0

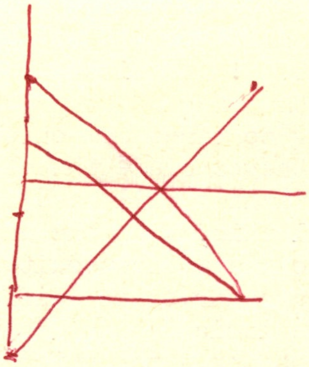
10 -10 0 2 1 1

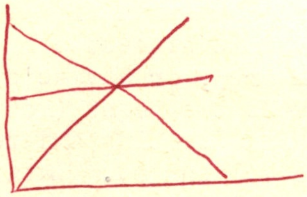
0 0 2 0 4 0 3 0

2 0 | 2 0 | 2 0 | -1 3 4 0 1.5 1.5

0 2 | 2 0 | 1 1 | 10 -10 10 -10 10 -10

-10 10 10 -10 0 0





$$\begin{matrix}1 & 0 & 1\\ 0 & 1 & 1\end{matrix}$$

$$\begin{matrix}1 & 0\\ 0 & 1\end{matrix}$$

$$\begin{matrix}2 & 0\\ 1 & 1\end{matrix}$$

$$\begin{matrix}1 & -1\\ -1 & +1\end{matrix}$$

$$\begin{matrix}2 & -2\\ 0 & 0\end{matrix}$$

Chapter VI

Becker

Smith, Fellner, Chipman } Chap. 5

What are they doing? Related to vagueness. Consider bids, sampling, p_o, etc. (Good)

What are they not doing: 1) minimax; 2) minimax regret; 3) Hurwicz; 4) Shackle;

~~Vagueness of~~ Uncertainty as to prob of Yellow/Black; show in payoff matrix. (assume only three balls; or two in each urn) (SAVAGE, R&S, dF, ALL ADMIT POSSIBILITY OF "VAGUENESS")

they are not: 5) using Rank II probs (Fisher, Good

6) using Principle of Insufficient Reason (Jeffreys, Knight) ~~(Chipman)~~

Moreover, they are not: 7) randomizing (Smith, Good, Savage) over reasonable dists

8) Picking one dist (Raiffa and Schlaiffer, Savage) (de Finetti).

9) being "indecisive" (violating P1).

10) lexico-graphic (Georgescu, Chipman) Fellner

~~11)~~ 11) Weighing vague probs toward 50:50

THEY ARE CONSISTENT WITH: 1) Hurwicz 2, Good 1, Bross

2) Hodges and Lehmann

3) My rule: Chipman example

4) "explains" p_o, P^o, etc. Fellner

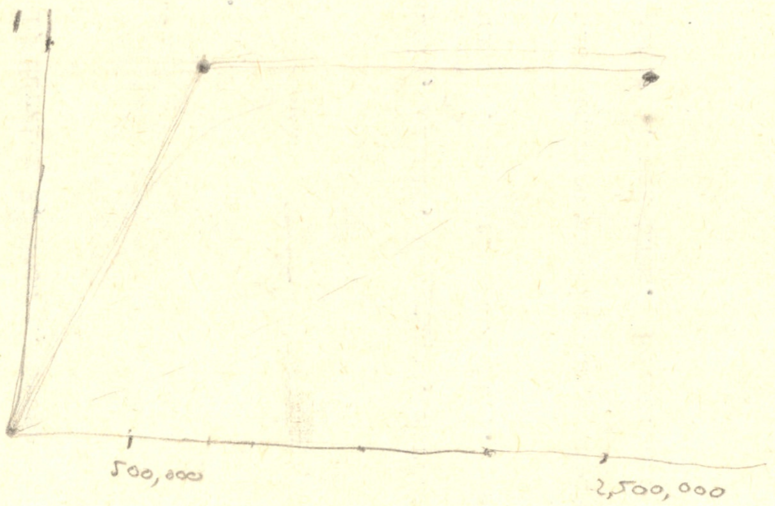
5) other rules as special cases (minimax, Shackle),

When will Sure-thing apply?

VII: Utility (including Rubin, Allais, Coherence, quote Marschak; Russian Roulette, problem of defining consequences, P2 vs P3 measuring utility). Long-run success. Randomization (vs Savage, Chernoff)

Cox, Smith: prediction.

VIII: how to measure vagueness in general (in our examples, we control info, "know" subjects' hypotheses); in general, ask them. Relation of p_*, t p^* to p_o, p^o.



$u(25,0;.1) = .1$

$.90$

$u(5,0;.11) > .1$

$.11 \cdot u(5) + .89 \cdot 0 > .1$

$u(5) > \frac{.1}{.11}$

$.909$

$.11\overline{).10.0}$

99

100

However, without pursuing that particular discussion any further, it is worth quoting certain of his remarks, because they lead naturally into consideration of some proposed postulates not yet considered. Raiffa addresses his comments to the second example presented in Chapter Five, the "three-color" urn experiment examined at length in Chapters Six and Seven (see Fig. 6.1). After concurring, on the basis of interrogating his graduate students in mathematical statistics and his classes at the Harvard Business School, that "most subjects choose act I over II and IV over III, thus violating the Savage Axioms," and that "many of these subjects are reluctant to change their choices when it is pointed out to them that their behavior is inconsistent with the Sure-thing Principle," he states that he would like to "undermine further ~~[?]~~ their confidence in their initial choices! I find the following argument is quite persuasive": ~~(it is not made clear whether his recalcitrant students find it equally persuasive):~~

> Suppose you register I > II and IV > III. I now offer you the paired comparison between the following two options: In option A a fair (unbiased) coin is tossed and act I is taken if heads appears, whereas act IV is taken if tails appears; with option B, heads leads to act II and tails to act III. In table form we get:

	Heads	Tails
Option A:	Act I	Act IV
Option B:	Act II	Act III

Fig. 8.7

> Now by strict dominance[1] option A is better than B *for you* since you prefer I to II and IV to III. So far there is no trouble but let's take a closer look at options A and B. The final outcomes of either

---

[1] Italics added.

If the law supposes that,... the law is a ass, a idiot. Bumble, Oliver Twist.

Notion of disagreement among "reasonable" opinions; group decision problem. Type II probs, etc.; but this would never generate observed behavior in these examples.

Note that rule does not violate notion that: actions should not contradict application of Bernoulli Principle to definite probability comparisons, where that leads to determinate choices; i.e., if same stakes are involved, and horse A is believed to be definitely "not less probable than B" then you should not definitely prefer to place the stake on B.

It can be formulated to avoid contradiction of coherence.

It cannot avoid contradiction of: P2; possibly P4; Rubin's axiom, Milnor axiom; Strong Independence Assumption. It has this in common with Allais example; it could lead to Allais behavior; or it could be "similar" to Allais example.

Effect on value of information, flexibility, insurance, all suggest similarity to Allais rationale: the "value" of a change in a certain payoff, or in a certain relative probability, is not independent (there is "complementarity") of the values of other payoffs, or the "absolute" probability of the affected subset of events (i.e., its probability relative to that of "all other" events).

Rule permits behavioral characterization of vagueness, relative vagueness; or, given "intuitive" characterization, permits testable hypotheses on the "effects" of vagueness on betting behavior; i.e., puts speculations of Knight, Savage, Ramsey in meaningful form (though not necessarily uniquely meaningful form).

Relation to Hrwicz rule: case of "complete ignorance" among reasonable hypotheses; relation to older Hurwicz rule, or minimax, in case of "complete ignorance" among all hypotheses (admitting all opinions as reasonable; represent "opinion" as a definite set of betting odds, or probabilities).

Hence, relation of this special case to Shackle.

EX-4-5194

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Hurwicz: ✓ relate to probability-utilities, uncertainty as to conditional probs

✓ Might think Savage would say meaningless, but he doesn't; question is,

✓ what to do when uncertain of proper distribution?

✓ Good also raises this question: probs known only within intervals, Y^o.

8 Randomly; arbitrarily (violate P1?) Raiffa and Schlaifer: pick one, no matter how vague. Why? So as not to violate P2.
(suppose first condition fulfilled)

— Jeffreys: pick uniform to "express ignorance." But does it? Commits you to 2:1 odds... Ambiguity as to distribution (Keynes?)

— Rather: suppose second condition fulfilled; after all, problem has been designed to produce "ignorance."

— But if these conditions are fulfilled, then other axioms imply Hurwicz rule; and measurement of . Answers "arbitrariness" of min and max.

— Shackle conditions.

— If they aren't fulfilled, Hodges and Lehmann. but with max, not just min.

add colors to urn

— Example: four colors. "Predict" your choices. Bayes approach unreasonable. Chipman

Coherence; perhaps = 1 unreasonable, or close to 1; and = 0 unreasonable (value of information reasons).

Leads to reluctance to gamble; need for "premium" to compensate for ambiguity; Value of flexibility; value of insurance.

~~Gambling personality.~~

anti-Fellner

~~Mapping~~

$\rho = 0$: $\rho = 1$

$\alpha = 0$ $\alpha = 1$

	α: 0	~~1~~	$\alpha > 0$
$\rho = 0$	Wald	~~"Maximax"~~	restricted Hurwicz (Shackle)
$>0, \neq 1$	Bayes	~~Bayes~~	Bayes
ρ	H+L		me

Gambling Personalities (Y^o; + U)

Wald, Hurwicz

Laplace

✓ sample; various in ρ (Chipman, Georgescu)

✓ Consequences :

McGregor

Weights as "probs affected by payoffs" — model of wishful thinking

gambling premium

Preface:

1. Such experimentation as there has been has concentrated on: objective probs, wide agreement, maximum information, precise; etc.

2. Bayesian theories purport to cover all uncertainty; I conjecture they are more suited to above case; I want to cover ambiguous cases: not just "complete ignorance."
Opinions matter, but they are vague

3. Theories for ambiguous cases should inspire experiements on those cases (Becker, Chipman, Fellner)

They may obey conditions of complete ignorance, for small Y^o. e.g., restrict Y^0 further in example. Three-way partition suchthat...

Even for large Y^o, conditions may be approximated. Much more reasonable

As approximation, are they more extreme assumptions than Jeffrey's Principle of Reason?

Note that Savage does without Principle of Insufficient Reason, but implications are just as powerful if first condition of ignorance is satisfied.

Note Knight assumptions. At least we can reject notion: do this because there is nothing else to do.

When conditions are not satisfied, but neither are axioms obeyed; vagueness;

Hodges and Lehmann; two forms.

✓ Restricted Bayes/Hurwicz principles

Four ball example: a) Bayes is unreasonable in both cases; (b) nothing but ~~xxxtxi~~ Hurwicz or Bayes/Hurwicz can explain both cases.

Chipman example.

Verbal rationale.

1. to act reasonably...

2. I wish to theorize about... Not necessarily whole area of Knight's "uncertainty"; he assumed definite opinions, personal probs:

3. Statistical decision theory; but limitations of rules, when dealing with prior probs; attempt to get rules independent of probs (Wald; de Finetti); yet need to take account of opinions as well as consequences, values. How to measure, express?

4. Hint in Knight: measure by behavior. But Shackle: people don't (in general area considered by Knight) act as if they assigned different numerical probs: example.

5. But betting behavior. Possibility of measurement is "by-product" of solution to problem of reasonable behavior.

6. Before analysing, criticising and modifying theory, consider relevance of "normative" principles to economics.

State Rubin's postulate

Examples: show my example violates...
Like Milnor, Blackwell and Girshick
Note: using utilities doesn't lead to obeying...

Insurance; comparison of expected values: Luce and Raiffa "critique"

Why "should" changing the payoffs affect answer?
Because it changes ambiguity of the set of ~~distributi~~ expectations; or
ambiguity of the probability distributions over payoffs

Like Allais: he refers to distribution over utilities; I was skeptical, since he assumes definite probabilities; yet his example...

My phenomenon is like his: premium on lack of ambiguity, just like premium on lack of risk.

Notion that preferences depend on "what other tickets you hold"

Utility axioms: if you violate Independence Axiom, can't measure utility when prizes are acts. But when are they not? Consider Savage, Marschak.

Savage on vagueness: when he would minimax regret. Argument applies to Hurwicz criterion as well. But

But, applying it to minimax regret: Russian Roulette.

What does it *mean* to *subtract* (or *add*: Milnor) vN-M utilities; i.e. what *are* "regrets"?

What, then, is "value of info"?

What does a "utility *difference*" *mean*, behaviorally, except ordering?
Would it be reflected, *practically*, in behavior?

State Rubin's postulate

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Like Milnor, Blackwell and Girshick
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Would it be reflected, *practically*, in behavior?

Bayes: $$\begin{array}{cc} \$S & \$0 \\ \$qS & \$qS \end{array} \rightarrow \begin{array}{cc} \$2S & \$0 \\ \$q2S & q2\$ \end{array} \rightarrow \begin{array}{cc} 0 & S \\ (1-q)S & (1-q)S \end{array}$$

$$\rightarrow \begin{array}{cc} -S & 0 \\ -qS & -qS \end{array} \nearrow$$

So if I will pay *less* for a stake S on E than on F, I must pay *more* for a stake S on $\bar{E}$ than on $\bar{F}$.

If I pay same for stake S on E as on $\bar{E}$, ~~the~~ and same for F as for $\bar{F}$, then I will pay same for S on E as on F.

$$\begin{array}{cc} 1 & 0 \\ q & q \end{array} \rightarrow \begin{array}{cc} 0 & 1 \\ (1-q) & (1-q) \end{array}$$

$q \qquad q\cdot 1 + (1-q)\cdot 0 = q$

$p\cdot 1 = q \qquad (1-p)\cdot 1 = 1-q$

$p = q$

With *multiple* goals, a *variety* of moves can look like *advances*; so trading, compromise, agreement is possible between individuals despite *conflicts* w.r.t. *certain* goals.

If utility is concave (prisoner), "insurance" may look "too expensive" unless it is at very favorable odds (say, \$10 increase in worst outcome = \$1 lowering in best); e.g. suppose that *best* outcome is just \$10 over margin of "survival" and lowering it \$10 (to improve worst by \$100, prob of worst = .9) means *no* chance of "acceptable" outcome. (0, 1) payoffs: Only "total peace" (or "total victory") is *acceptable*; no increase in other outcomes can compensate for *any reduction*.

When some measure (arms control) "compromises" *both* total peace and total victory, both sides are against it. Note attitudes opposing "compromise": total lack of *substitutability*, multiple goals.

If $\alpha > \frac{1}{2}$ and a book can be made against X in utility, opponent can't necessarily make money on X, if his utility is convex.

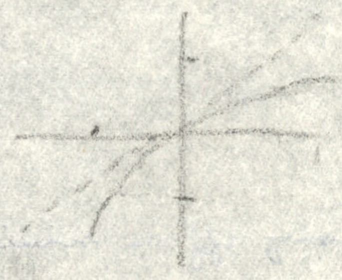
Suppose $\begin{matrix} 2 & -1 \\ -3 & 1 \end{matrix}$

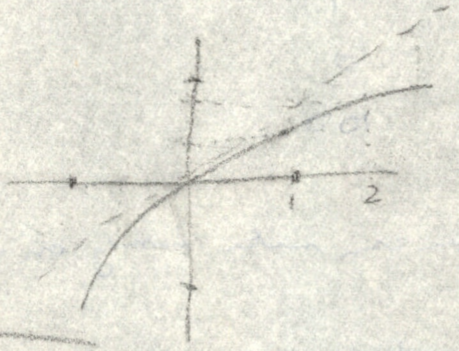
$$\begin{matrix} \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 \\ -\frac{4}{5} & \frac{1}{5} \end{matrix}$$

then take both and lose

$\frac{1}{2}$

Money	U	M	U
(2)	$\frac{2}{3}$	$(-\frac{1}{4})$	$-\frac{1}{3}$
$(-\frac{1}{2})$	$-\frac{3}{4}$	$(\frac{1}{2})$	$\frac{1}{4}$





Assume expectation is a continuous smooth function of the dist; then small differences in the determination of Y_{min} & Y_{max} won't matter.

---

assuming 0, 1 payoffs

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$$

is assuming "success or failure," "~~statistician's~~ scientist's payoffs."

Risk fn. becomes the probability of success. ~~[illegible]~~

My subjects have both an estimation problem and a decision problem (though Savage doesn't distinguish; he infers estimates from decisions; I distinguish;

~~estimates don't [illegible]~~

My problem: how do people act on the basis of internal estimates? Savage: if they are not foolish, they act as if they had point estimates — which you can infer from their choices. Me: no they don't.

Try to make money off a minimaxer:

$$\begin{matrix} 100 & -1 \\ 0 & 0 \\ -1 & 100 \end{matrix}$$

It's a maximaxer you can make money on:

$$\begin{matrix} -100 & 1 \\ 1 & -100 \end{matrix}$$

Savage: person knows his mind about actions, has a) preferences, and b) rules of consistency. Koopman: person knows his mind about probabilities, a) assigns intervals, b) rules of consistency.

Savage's rules of action ⟹ stronger estimates and rules than Koopman's, lead to point estimates. But now a) may not feel foolish if he doesn't follow them, b) may not be necessary if he doesn't [illegible]

Savage:

"Coherence" argument is really used by those who believe in *linear utility*: at best, (like Savage) for *small sums*.

Requirement that ~~stakes~~ p be fixed for *all* S, positive or *negative*, is a *linearity* requirement; which Savage thinks reasonable if deviations are *small*.

i.e. variation of S in ~~a~~ close neighborhood of 0 should not affect p; e.g. p should be the same if $S = 1$ or $S = -1$, if unit is *small*; should be no *kink* at 0.

$p_* = f(a, b; E_1, E_2 \dots)$ should be *smooth* function of a and b; should be constant for $(0+\epsilon, 0-\epsilon)$; $(0-\epsilon, 0+\epsilon)$ for ϵ sufficiently small.

How should you act when (a) you know utilities, and (b) you have only an interval estimate of probabilities (your interval estimates being "consistent" in Koopman's sense)?

Suppose someone says: "Minimax with respect to your reasonable estimates." (Good, Hurwicz, Hodges and Lehmann, Bross). This gives determinate advice. Suppose you follow it.

But then Savage says: You are violating my postulates!

So what?

1) Don't you feel foolish?
   Not really.
2) Don't you want to change some of your decisions? Not really; why should I? And which ones would I change?
3) A book can be made against you; you are incoherent! NO; you are mistaken.

(Axioms are not necessary to coherence in this sense, despite de Finetti and Savage).

Suppose Savage says: here is another rule to follow, which won't lead to violations of my axioms; I would feel better if you used it. (And easier to describe your OK.(behavior; and you can always answer, "Which event is more likely?"

But you can then be shown that you must, in effect, ignore ambiguity; you must act alike in the following situations....you must make the following bets. Can you live with this?

You canalways state, "Which event is more likely?"; but now you can no longer meaningfully (in terms of behavior) state: I am less sure of this judgement than of that; this situation is less clear-cut than that; this estimate is less reliable than that.

Note de Finetti, p. 220, Berkeley: "one's own belief, which cannot be unknown to oneself..." or "any belief concerning a priori probabilities corresponds to a distribution on the set of possible probabilities. 223

Savage: people act "as if" they had a complete ordering of probabilities, ~~whenxwhen~~ even when their statements might establish only a partial ordering (i.e., even when they might often say they "couldn't compare" two probabilities, or two events(.

Me: people may act "as if" they had only a partial ordering of probabilities --i.e., they may act inconsistently with assumptions that they max expected utility on basis of complete ordering--or may "reveal" only a partial ordering--even when their statements might reveal a complete ordering (e.g., in terms of "best guess" probs, but ignoring relative vagueness((, as well as when their statements revealed only partial ordering.

I wish to make certain distinctions or hypotheses which Savage and followers tend to assert are "meaningless." I could simply state them and later show them to be meaningful in terms of Savage's own operations; but instead I will open with a discussion of operations in terms of which they are clearly meaningful.

Pratt:

1. Assume person offering the bets knows no more than you (so the offer conveys no info about the actual contents of the urn).

Option A: I or III: $v = v(I) = \frac{1}{4}$

Option B: II or IV $v = v(II) = \frac{1}{4}$

	H	T
3,4	A	B

$v = \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{4}$

Suppose bet is drawn at random from

$$\begin{array}{c|cccc}
 & R_{I} & B_{I} & R_{II} & B_{II} \\
\hline
I & 1 & 0 & 0 & 0 \\
II & 0 & 1 & 0 & 0 \\
III & 0 & 0 & 1 & 0 \\
IV & 0 & 0 & 0 & 1
\end{array} \qquad III \quad 1 \quad 0$$

		EV
Take I if I offered, III if II offered	$\frac{1}{2}\cdot v(1\ \ 0) + \frac{1}{2}\cdot\frac{1}{2}$	$\frac{1}{4} - \frac{3}{4}$
Take II if II offered, III if I offered	$\frac{1}{2}\cdot v(0\ \ 1) + \frac{1}{2}\cdot\frac{1}{2}$	$\frac{1}{4} - \frac{3}{4}$
Take III if I offered, III if II offered	$\frac{1}{2}$	$\frac{1}{2}$

$$\begin{array}{ccccc} R & 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ & 1 & 0 & & \end{array}$$

$$\overbrace{Y \quad B}$$

$$\begin{array}{ccc} & H & \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \qquad \begin{array}{cc} \multicolumn{2}{c}{T} \\ 0 & 2 \\ 0 & 2 \end{array}$$

$$\begin{array}{ccc} \multicolumn{2}{c}{60} & 30 \\ \multicolumn{2}{c}{\overbrace{\qquad\qquad}} & \\ R & Y & B \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & & 1 \\ & & 1 \end{array} \qquad \begin{array}{ccc} \multicolumn{2}{c}{\overbrace{\qquad\qquad}} & \\ R & Y & B \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}$$

Rubin

$$\begin{array}{ccc} & H & \\ R & Y & B \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \qquad \begin{array}{ccc} & T & \\ R & Y & B \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{array} \qquad \left.\begin{array}{ccc} 2 & 0 & 0 \\ 0 & 2 & 0 \end{array}\right\} \begin{array}{ccc} 0 & 0 & 2 \\ 0 & 0 & 2 \end{array}$$

$$\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}$$

4

I > III
II > IV

III = IV but I
(III, IV) > II

Heads Tails III > II

$100 I

Heads Tails

$\frac{1}{2} \cdot Y_0 + \frac{1}{2} \cdot Y_{min}$

$\frac{1}{2} \cdot p + \frac{1}{2} \cdot (1-p)$ ρ $\frac{1}{2} \cdot p + \frac{1}{2} \cdot (1-p)$

~~$\frac{1}{2} \cdot \frac{1}{2}$ +~~

$\frac{1}{4}$

Heads-Red	Tails-Black	Heads-Black	Tails-Red
100	100	$0	$0

Heads		Tails	
Red	Black	Red	Black
100	0	0	100

$\frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2}$ $\frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2}$

$p \quad (1-p)$

1

Red		Black	
Heads	Tails	Heads	Tails
$100	$0	$0	$100

$p \cdot p($

A

$Y_0 = \frac{1}{2}, \frac{1}{2}$
H.

0 1 1

$\frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2}$

$100 $0

Red, Heads, Black, Tails Black, Heads; Red, Tails

$100 $0

		H			T		
				R	Y	B	
I	100	0	0	0	0	0	I > II
II	0	100	0	0	0	0	
III	0	0	0	0	100	100	
IV	0	0	0	100	0	100	

I > [illegible]

V	100	100	100	0	0	0	V
VI	0	0	0	100	100	100	
VII	100	0 ~~0~~	0	~~100~~ 0	100	100	
VIII	0	100	0	100	0	100	

Given VII ≐ VIII ≐ ~~V ≐ VI~~, III > IV, prove II > I

I	100	0	0	0	0	0	
II	0	100	0	0	0	0	
	100	0	0	100	0	100	P2
	0	100	0	100	0	100	
	100	0	0	100	0	100	P1
	100	0	0	0	100	100	
	0	0	0	100	0	100	P2
	0	0	0	0	100	100	

A: $\frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{2}$

B: $\frac{1}{2}\cdot(\frac{?}{\cdot}) + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot(\frac{2}{3} - ?) = \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{2}$

H			T		
R	Y	B	R	Y	B
100	I 0	0	0	IV 100	100
0	100 II	0	100	0 III	100

100 0 0

Red/Heads Yellow/Heads Black

H			T		
R/H	Y/H	B/H	R/T	Y/T	B/T
100	III 0	100	0	II 100	0 ~~10~~0
0	100 IV	100	100	II 0	0 ~~10~~0

$\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

$\frac{1}{2}\cdot ? + \frac{1}{6} + \frac{1}{2}(\frac{2}{3} - ?) = \frac{1}{2}$

A $\frac{1}{6} + + \frac{1}{2}\cdot(\frac{2}{3} - ?) + \frac{1}{3} = \frac{1}{2} + \frac{1}{2}(\frac{2}{3} - ?)$

B $\frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}(\frac{2}{3} - ?) = \frac{1}{3} + \frac{1}{6} + \frac{1}{2}(\frac{2}{3} - ?)$

A $\frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}(\frac{2}{3} - ?) + \frac{1}{2}\cdot(\overset{?}{\cancel{\frac{3}{3}-?}}) = \frac{1}{6} + \cancel{1}\cdot\cancel{(\frac{2}{3} - ?)} = \frac{1}{2}$

B $\frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{2}$.

A		$A \cup B$	B		$\bar{A} \cap \bar{B}$
Red/Heads	Yellow/Tails	Black/Tails	Red/Tails	Yellow/Heads	Black/Heads
100	100	100	0	0	0
0	0	100	100	100	0

Red/Heads	Yellow/Tails	Black/Tails	Black/Heads	Yellow/Heads	Red/Tails
100	100	100	0	0	0
0	0	100	0	100	100

$\frac{1}{6}$	$\frac{1}{2} \cdot ?$	↓	$\left(\frac{2}{3} - ?\right)\frac{1}{2}$			$= \frac{1}{2}$
100	100	0	100	0	0	
0	0	0	100	100	100	
			$\left(\frac{2}{3} - ?\right)\frac{1}{2}$	$2 \cdot \frac{1}{2}$	$\frac{1}{3} \cdot \frac{1}{2}$	$= \frac{1}{2}$

$\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

$\frac{1}{3} \cdot \frac{1}{2} \quad ? \cdot \frac{1}{2} \quad \left(\frac{2}{3} \cdot ?\right)\frac{1}{2} \qquad \frac{1}{3} \cdot \frac{1}{2} + ? \frac{1}{2} \quad \left(\frac{2}{3} - ?\right)\frac{1}{2} = \frac{1}{2}$

$(R/H \quad Y/T \quad B/T) \overset{?}{—} (R/T \quad Y/H \quad B/T)$

$(R/H \quad Y/T \quad B/H) \qquad (R/T \quad Y/H \quad B/H)$

$\frac{1}{3} \cdot \frac{1}{2} + ? \frac{1}{2} \quad \left(\frac{2}{3} - ?\right) \cdot \frac{1}{2} \qquad \frac{1}{3} \cdot \frac{1}{2} + ? \frac{1}{2} \quad \left(\frac{2}{3} - ?\right)\frac{1}{2} =$

$\frac{1}{6} + \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{2} \qquad \frac{1}{6} + \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{2}$

E $\quad\quad$ $\bar{E}$

$-S$ $\quad\quad$ 0

$p(-S)$ $\quad\quad$ $p(-S)$

I $\quad$ $p^*(-S) - (-S)$ $\quad\quad$ $p^*(-S)$ $\quad\quad$ ≥ 0

II $\quad$ 0 $\quad\quad$ 0

What is greatest $p \ni \; I \geq II$?

$S(1-p^*)$ $\quad\quad$ $-p^*S$

$p(S)$ $\quad$ What is the lowest p^* for which $\; I \; p \; II$?

$-S$ $\quad\quad$ 0

$p(-S)$ $\quad\quad$ $p(-S)$

What is $p^* \ni \; V((p(S)) = V(I)$

$(p) = \dfrac{S - p(S)}{p(S)} = \dfrac{1-p}{p}$ $\quad\quad$ α $\quad\quad$ β $\quad\quad$ $\dfrac{\alpha - \gamma}{\beta}$

γ $\quad\quad$ γ

degree of belief: $\dfrac{1-p}{p}$ $\quad\quad$ $\dfrac{-S * (p(-S)}{p(-S)}$ $\quad$ $\dfrac{1-p}{pS}$

I Introduction: relevance of normative criteria to economic theory; "tests" of normative axioms. Goal: normative criteria for decisions under partial ignorance.

II. Given various circumstances (of uncertainty), what to do. Need to classify circumstances, since rules may differ. Payoff matrix; a representation of problem compatible with different circumstances, or different views on meaningful distinctions.

~~Riskxx~~ Circumstances when probabilities may be meaningful: risk, a priori calculation. Rules that may be based on calculations.

When probabilities are not "given" objectively: rules that are meaningful, may be useful: Wald minimax; minimax regret; Hurwicz. Counterexamples; they are not adequate normative criteria, even for a particular individual; possibility that they apply in special cases; problem of specifying these. Laplace rule. (arbitrariness counterexample). COMPLETE IGNORANCE; Arrow proof. But for partial ig;?

Rules that assume measurements of subjective belief: a) suppose partial
Koopman, ordering;(Keynes); suppose some appraisable (Koopman); c) suppose interval
Good, p.O.; measurement of probs (Hurwicz, Hodges and Lehmann, Bross) (Why? When meaningful?
weight of arg. e)Suppose point measurement of probs by bets. d) Shackle Will bets permit consistent measurement? WON'T WORK WITH MONEY BETS. (Bayes, de Finetti). Utilities? d) suppose some other choices known ; are there rules of consistency from which to derive present choice? Rules for generating counterexamples to other rules: applications of other rules that violate these rules. (~~Hx~~ Savage, Ramsey, de Finetti.

Their rules allow us to prescribe choice from other choices; infer probs for all events; rule out all other rules, including H&L, in fact everything but maxing utility on given probs. 0) lack of testability, data.

Earlier approaches had one or more defects:/a) not applicable to all cases; (b) known counterexamples; possible to find case for every person that would bw counterexample; c) even before counterexamples discovered, feeling of arbitrariness.

Few or none persistent counterexamples to Savage axioms (Dreze restriction on application) . ((To find some, look for cases of "equal probs"...because Savage lumps cases of known, equal probs together with cases of "ignorance"; if behavior is different for latter, it will show up here conclusively, though we are not interested just in these cases)).

III Savage axioms; show "necessity",for inferring probs. Examples

IV Counterexamples. to P2 P2

V Explanation in terms of non-appraisability (Koopman), H&L, Hurwicz decision rules. Pragmatic effects. Smith: bets

VI Implications for other axioms; P4, dominance, anti-convexity. Rubin

Is Nature malevolent?
VII Coherence; rules out maximaxing, but not above rule. Problems with coherence test. Rule of Long Run Success; doesn't apply. (unless definite probs.
"social advantages" of wishfulness; empirical relevance; Keynes, Knight; innovation.
CHIPMAN VIII Relate to Knight, Shackle; Keynes (Prob, GT).

IX Relevance to: systems analysis; game theory; value of information; incentives to forecasters; insurance vs. flexibility; gambling personalities; wishfulness and innovation (Knight) (Keynes). R+D.

1. ~~Bern~~ The Bernoulli Principle. As hypothesis, theorem, recommendation. In latter form: as hypothesis on desired, reflective behavior; ~~a principle you would~~ ~~The~~ Even in general, existence theorem form, it has operational content; hyp: that you will not wish to act so as to contradict the existence of...

Note relation of theory to measurement: if one variable is "given"...

2. ~~If~~ Two forms of theorem: if probs are"given", ~~you~~ action contradicting existence of utilities would violate vN-M axioms; you wouldn't want to do that (Marschak axioms, etc.). But this would cover, at best, behavior when probs are "given"; and in strict sense, they never are (can't strictly control information available to person). If utilities were given (money), action contradicting existence of probs would violate de Finetti, Bayes, principle of coherence plus special axiom.

But very generally, certain patterns of choice in certain situations would contradict existence of any utilities or probabilities whatever. ~~If you~~ Abstract from utility problems (Borel, Ramsey) by having only two outcomes: real test of notion that only relative likelihood and relative preference are involved.

3. Real event: bet on Dick vs. Jack. If bets were in money, Smith behavior would not contradict existence of probs; but if bets were in utility (i.e., numbers that worked for some events), it would. I conjecture that this will happen; will give examples. Contradiction of Bayes-de Finetti approach. Coherence.

4. If it were never true that intuition did not provide immediate answers in complex situations, and if no contradictions of intuitions ever appeared, there would be no need for theory (~~For~~ Raiffa point); but granted that a theory is needed, the test of a good, useful theory is that one is prepared to accept, on reflection, ~~to~~ its implications in complex situations, and that it is "easy" to resolve contradictions in a way that is acceptable, intuitively compelling, or at least in a way that does not do violence to one's honest, definite opinions. (otherwise, ~~theory~~ application of theory merely discovers or creates "new problems" in the attempt to solve given ones).

5. START WITH: Pre-Bernoulli theory: math expectation, Bayes, de Finetti; definition of prob in terms of odds; coherence, fair betting; principle as direct normative intuitive principle, not terribly plausible (not powerful motive to change behavior).

~~Bernoulli counterexample Smith~~ Bernoulli counterexample; effect on measurement of probs (Smith example; Savage remarks, interms of money).

Bernoulli theory:

Smith examples would be counterexamples, if in terms of utility; but need measurement of utility. (Conjecture that they will hold, when we measure utility).

But Bernoulli theory not only escapes old counterexamples (e.g., his own); it has been shown to follow as theorem from other axioms that make a more direct appeal to intuition; seem "more likely" to characterize reflective behavior.

6. Action that contradicts Bernoulli theorem in "simple" cases shows violation of ALL sets of axioms, theories, that lead to this theorem: Suppes, etc. SUCH PEOPLE, NEED AN ALTERNATIVE THEORY (AND A "BETTER" THEORY FOR THEIR PRESENT BEHAVIOR IS LIKELY TO HAVE USEFUL IMPLICATIONS FOR PREDICTION AND MEASUREMENT), AT LEAST UNTIL SUCH TIME AS THEY ARE PERSUADED THAT THEY MUST "RE-EDUCATE" THEIR PREFERENCES, A natural heuristic assumption is that they are seeing a difference that makes a difference: between those situations in which they obey axioms and those in which they don't.

7. Examples:..... Intuition, myself, Raiffa, Smith conjectures, Chipman subjects, Becker subjects (so at least theory isn't contradicted by unreflectiv behavior)

8. What will not explain the examples... Chipman

9. What will: vagueness, Koopman-Good; Hurwicz and Hodges and Lehmann

Chapter VI

Now we have a rule; let's see where it leads. In particular, to test its reasonableness, let's deliberately seek out violations of the postulates and their implications. If anything will discredit it, that will.

We have already seen violations of the P2 and P4. These tests of reasonableness require a minimum of prior observations, data; but they detect only "gross" violations, requiring very special sorts of alternatives for test.

If more data on subject's choices are available, then a wider variety of alternatives will provide tests of consistency; there is more for him to be consistent with, his choices are more severely constrained by the theory. We can test some rather subtle propositions that may not be intuitively obvious in themselves, but that follow logically from the body of the axioms. (To violate a postulate is to contradict a proposition that has been selected as being particularly intuitively persuasive) (and testable on minimum observations; basic notions of consistency).

Since our rule violates postulates, it is bound to violate many propositions deducible from the axioms. But some of these are particularly interesting, since they are often regarded as as intuitively compelling as the postulates themselves; in fact, several of them have been used as postulates in other approaches.

In particular, the utility axioms are deducible as theorems from the Savage axioms. These apply only in circumstances when probabilities are "known" and specified; in original applications, these were assumed to be "objective" frequencies; or, we can imagine that subject has been found to obey Savage postulates in certain cases and we have measured his probabilities.

(On the other hand, we can assume utilities "known" and derive propositions about probabilities: Bayes, de Finetti, coherence arguments).

In other words, we can assume probabilities known, and test propositions about lottery tickets, risk prospects (test utilities);

Or we can assume utilities known, and test propositions about odds, or "revealed probabilities."

Disadvantages: propositions may be less intuitively compelling; prior observations needed, or assumptions as to the results of observations not performed.
Advantages: propositions older, more familiar; though prior observations needed, conditions need be less special...we can get a "test" in a situation which would not test the postulates directly, e.g., bets involving sure-things, prices to be paid for bets, bets with several events and different outcomes.

(Another disadvantage: can't trace conflicts to violation of a particular axiom, since propositions derive from several axioms).

We will find that our decision rule leads to violation of the utility axioms; from a descriptive point of view, this means we can't assign von Neumann-Morgenstern utilities to ambiguous strategies; and since a number of prizes may be regarded as ambiguous strategies (Savage), this may suggest a large area of inapplicability of axioms, a fetile source of violations. Means we must test to be sure that utilities can be assigned to particular consequences.

$\pm$ S 0

Π $p_0 S$ $p_0 S$

$-S$ 0

$p^0(-S)$ $p^0(-S)$

$2S+b$

$a > 0$

$\Pi'(2S+b)$ $(2 0+b)$

Π' $p(2S+b)$ $p(2S+b)$

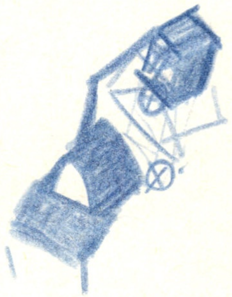
$a > 0$

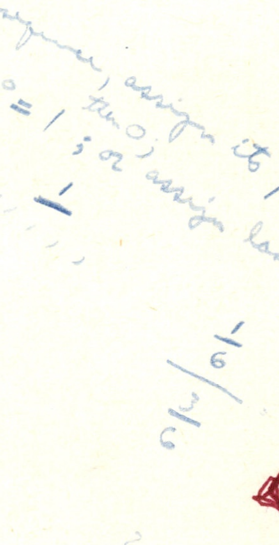
$\rho = 0$ $\alpha = 0$

To determine weights; a) start with worst consequence, assign it p^0; then next worst, assign it p^0; till $\Sigma^k p_i^0 = 1$; or assign last prob so that $\Sigma p_i^0 = 0$

a (b) start with best, assign p_0 (e.g. 0), then 0; ...

$\frac{1}{6}$ $\frac{1}{3}$ $\frac{6}{4}$ $\frac{1}{6}$ $\frac{3}{6}$ $\frac{1}{3}$





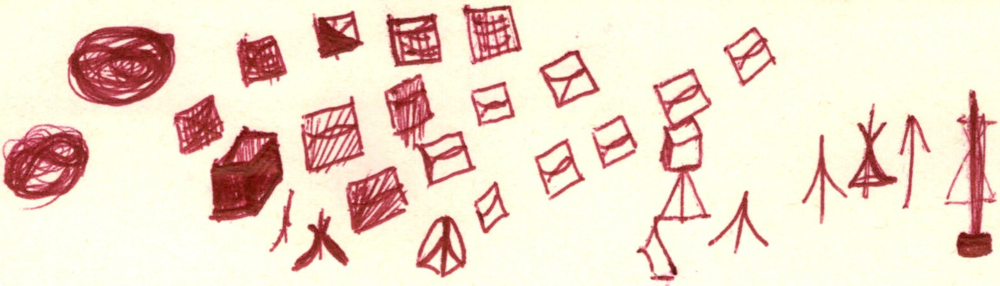
Use my index on money outcomes)
to produce utilities?
(even with definite probs)
NO; but somehow "double-weight" bad possibilities.
Why is it "reasonable" to have curved vN-M utility? (Allais would say: it isn't.)
(apply "definiteness" assumption to money outcomes: you get linear utility).

$\frac{1-p}{p}$

$p \cdot g + (1-p) \cdot (-1)$

0 $pg - 1 + p = 0$

$g = \frac{1-p}{p}$



It can, however

But with Hurwicz: show values Contrast to "

Show diagram: HOW EXPLAIN BAYESIANS? PRINCIPLE OF INSUFFICIENT REASON?

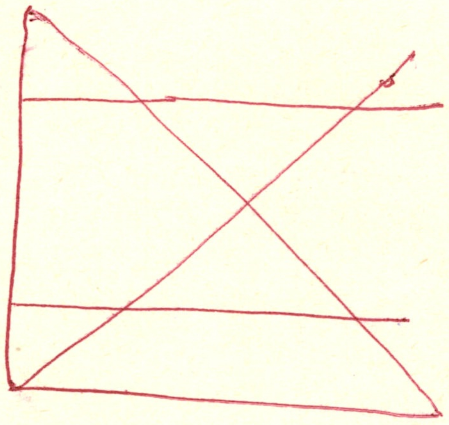
Interpret flat strategies as "insurance"

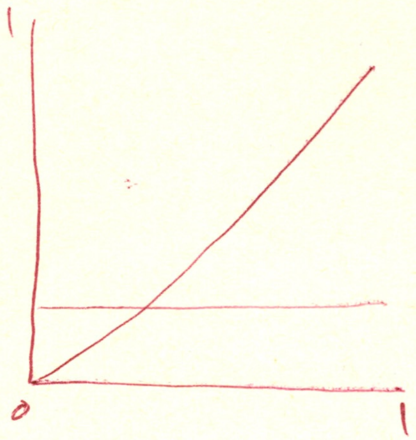
quote: insurance can cost too much. Bayes/minimax: desire for payoffs insensitive to ambiguity; (this is unreasonable [redacted] "in general"; reasonable if unambiguous payoffs are "good").

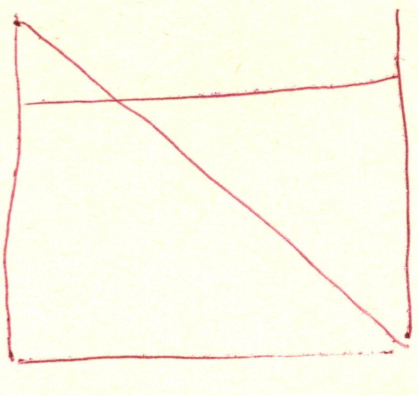
Why must be less than 1/2. Effect of weighint unfavorable; quasi-pessimism (thus, model for pessimism or wishful thinking); but don't go too far.

Quotes:

Raiffa; me







1

2

$\frac{1}{4}\Delta M = \frac{3}{4}\Delta m$

1 0

$\frac{3}{4}$ $\frac{3}{4}$

1 -3

$\frac{3}{4}$ $\frac{1}{2}$ $\frac{1}{2}\cdot\frac{1}{2}+$

$\frac{1}{8}+\frac{1}{6}$ $\frac{1}{24}$

7

$\frac{3}{5}+\frac{1}{6}$

$\rho =$ ~~$\frac{1}{2}\cdot\frac{1}{3}$~~

$\frac{1}{4}\cdot a+\frac{3}{4}\cdot\left(\frac{1}{3}M+\frac{2}{3}m\right)$

$\frac{3}{16}+\frac{3}{4}\cdot\frac{1}{3}$

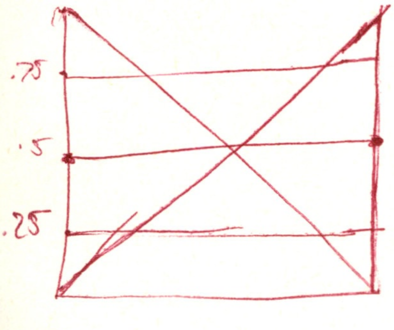
$\frac{1}{4}\cdot\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{3}$

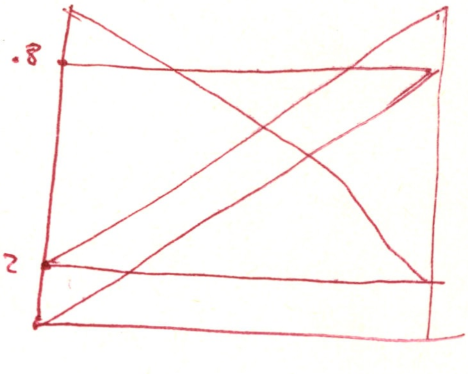
$\frac{1}{16}+$

				$\rho = 0$	$\rho = \frac{1}{4}$	$\rho = \frac{1}{2}$	$\rho = 1$
	est_x	max_x	min_x	$\alpha = \frac{1}{3}$			
I ~~III~~	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
II	$\frac{3}{4}$	1	0	$\frac{1}{3}$	$\frac{7}{16}$	$\frac{13}{24}$	$\frac{3}{4}$
III	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
IV	$\frac{1}{4}$	1	0	$\frac{1}{3}$	$\frac{5}{16}$	$\frac{7}{24}$	$\frac{1}{4}$

$$\begin{array}{ccccc}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1
\end{array}$$

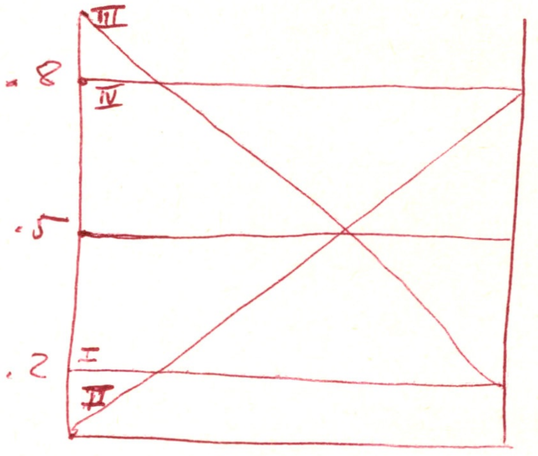
$$\begin{array}{ccccc}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 \\
\hline
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0
\end{array}$$





P_o ([illegible]$_{II}$) ≥ P_o(

$\frac{p_o}{q_o}$



Keynes' Point of view: p.o., weight of argument.

Knight: confidence ___

Good, Koopman: s-scales, intervals

(don't say, like Keynes, prob 'doesn't exist" but that it is known only within wide interval).

How to act?

Meanwhile, Savage/Ramsey try different measurement = bets

assume person tries to max prob. of better event

S 0

0 T

No

S 0

p^S p^S

No

Yes 1 0

0 1

1 0 0

0 1 0

1 0 1

0 1 1

} P2 assumes *additive* property

(*Sherman*: $A > B \Rightarrow \bar{A} < \bar{B}$

Vary weight of *argument*; *do* people obey axioms?

No. But they *do act* as if probs. had p.o.; intervals.

S. *Axioms* imply no expression to vagueness. OR *differences* in personality in response: α, ρ.

Intolerance of ambiguity: high ρ as personality characteristic — ~~or the~~ possibly combined with pessimism or optimism.

Insurance tendency assumes willingness to see, respond to, admit ambiguity. Intolerant types can't admit, hence can't explicitly insure.

Instead of stated comparisons to an n-scale, Savage/Schlaiffer assumes comparison of bets

But won't Koopman phenomenon turn up?

$$\begin{array}{cccccccc}
A & 1 & 2 & 3 & 4 & \cdots & 100 \\
1 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & 1 & 1 & 1 & \cdots &
\end{array}$$

$$\begin{array}{ccccccccccc}
\multicolumn{2}{c}{\overbrace{R_I \quad R_{II}}^{100}} & 1 & 2 & 3 & \cdots & & & & & 100 \\
1 & 0 & 0 & 0 & \cdots & & & & & & \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & \cdots & & & \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & & & \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & & & & \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & & & & \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1\ 1
\end{array}$$

Savage rules this out, by P2 (i.e. preferring to bet on n-scale, both on r vs. A and n−r vs $\bar{A}$

$$p_* < \frac{r}{n} < p^*$$

2

Chapter 2

Bernoulli

problem of measuring utility and probability; ~~if~~
Suppes and Davidson: at least these variables; hypothesis that only these variables
~~if probs are "known": von Neumann and Morgenstern utilities~~

if utilities are known: e.g., if utilities are money: Bayes. (de Finetti)

but Bernoulli's own objections.

Or if probs are known: e.g., "objective"; von Neumann-Morgenstern utilities.

Ramsey: measure prob=$\frac{1}{2}$, then measure utilities, then measure probs

Savage: measure probs, then utilities.
Can apply approach to cases with simple payoffs: 1,0.
Under what conditions will this lead to definite probability relationship over events? Postulates specify; and are asserted to be normatively compelling.

When probs are "not known": e.g., not "objective" minimax, minimax regret, ~~[illegible]~~ Shackle
Keynes, Koopman, Good; intuitive, verbal, consistency. But ~~incompl~~ partial ordering. (also, question whether these would be "appropriate" probs for Bernoulli hypothesis, any more than "money" is appropriate for hypothesis, or even "utility" as revealed by intuition and questioning.)

Incentive to measure probs in a way that will yield: a) precise, definite complete ordering; b) justify Bernoulli hypothesis, either descriptively or normatively. ~~Betting behavior~~ (Both Ramsey and Savage assume prob relationship must be complete ordering). Also, feeling that behavior is more "reliable," more "relevant" basis for measurement.

Theory: In choosing among actions, ~~you~~ which offer the same two outcomes, you will choose the action with the ~~more~~ greater likelihood of the better outcome. Hence we can infer likelihoods; and these will be the same no matter on which events we stake the prizes.

My theory: choose the one with the higher lower probability.

From choices among ~~[illegible]~~ acts with prize on a given set of events, you can't always predict choices from acts with prizes on different events.

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}$$

$$\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}$$

However, if you know: a) intuitive probs

or b) utilities

or c) definite probs

you can measure upper + lower probs for each event, and predict all choices.

$$\begin{array}{cccc} * & E & F & E \\ 1 & 1 & 1 & 1 \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & \frac{2}{3} \end{array}$$

$$\begin{array}{cccc} E & F & \bar{E}\cap\bar{F} & \\ 1(\frac{1}{3}) & 0(\frac{1}{2}) & 0 & \frac{1}{3} \\ 0\,\frac{2}{3}) & 1(\frac{1}{4}) & 0 & \frac{1}{4} \\ 1\,\frac{1}{3} & 1\,\frac{1}{4} & 0 & \frac{7}{12} \\ 0 & 0 & 1(0) & 0 \end{array}$$

$\frac{7}{12}$

$$\begin{array}{ccccc} & E & F & & \\ & -1^{(\frac{1}{3})} & 1^{\frac{1}{4}} & 0 & -\frac{1}{2} \\ & 1(\frac{1}{3}) & -1(\frac{1}{2}) & 0 & -\frac{1}{6} \\ \hline \mathrm{I} & -1 & 1 & 0 & \mathrm{I} > \mathrm{II} \Rightarrow F_* > E^* \\ \mathrm{II} & 0 & 0 & 0 & \end{array}$$

If intuitive probs are appraisable, "betting probs" may not be, or may not correspond.

e.g., you may maximize a function of mean + variance of utility (violate Sure-thing Principle); or of money.

Or, you may violate utility axioms.

If we just want to predict forecaster's behavior, enough to observe his bets: b_*, b^*.
But to use his forecasts, or check (against his betting behavior), we want to know p_*, p^*, "best guesses.", ρ, α.

Ask also: not just, "What is smallest interval for which you will give 20:1, but: What is smallest interval for which you will give 10:1; 5:1; 2:1; 1:1? The difference between interval on which I will give 20:1 and smallest sub-interval on which I will give 2:1 may be negligible.

i.e. even when 95% confidence interval is of moderate size, smallest 50% confidence interval may be almost as large (though with rectangular dist, it should be about half the size, and with non-uniform dist, even smaller. Violates axioms.

Likewise, whenever smallest 50% confidence ~~level~~ interval given A is more than half of interval A (whatever confidence level of A), violation of axioms.

→ ambiguity within A.

If forecaster is type that doesn't "see" ambiguity, he may not be as useful to us; advantage of having several forecasters, who may disagree (though must allow for different payoffs, decision rules, etc.).

Above betting behavior implies ambiguity; though not vice versa.

Virtues of theory of belief vs theory of decision or action.

Intuitions

My ideas have appeared in Good, Koopman, Smith in their theory of ~~decision~~
judgments, beliefs, probs; but they have not noted conflict with theory of
decision, or "rational behavior," nor tried to reconcile the two.

Hurwicz + H+L have my theory of behavior, but same is true w.r.t. conflict
with Savage theory.

None present (my) examples in which conflicts emerge, none defend
(my) theory normatively, nor support it "empirically." They mention it at
most as a possibility (though do associate it with ambiguity, lack of
precision). Other possibilities Good + Smith mention (or Chipman) do not violate Savage axioms.

Chipman presents empirical results supporting my theory, without
explaining.

Thus, H, H+L, G all propose a possible decision rule, for certain
cases. I argue that people do obey that rule in those situations,
even though they know it violates Savage axioms; hence, S theory is
not normative for them.

3. I get *partial ordering* of probs (like Koopman, Keynes; vs. Savage). My S's *should* obey Koopman's axioms; translate these into axioms on *choice*.

4. A/W "solution" is really *minimax*. *Insures against* Type I Uncertainty (Klein); ignorance on prob. dist. with high variance. ~~But ignores~~ (Buys Type I Flexibility; Mixed force. May not look "flexible" in ordinary terms; armor, life insurance. Invulnerability; insensitivity to certain events; keeps later options from being *foreclosed* by the impact of certain events, i.e. prevents death or ruin (one can stress ~~that~~ the badness of death or ruin in itself, or the effect of death or ruin upon one's *subsequent* prospects. Latter interpretation ~~sta~~ makes "insurance" look like means to "flexibility." Problems: a) Is "adequate" insurance available, and is it clearly "cheap enough"? b) Would *much* better solution be possible with better-available-info?

Klein: wants adaptiveness, lack of commitment, and program of buying *info* to get Type II Flexibility; a response not to Type I Uncertainty but to the *prospects of improving one's information* and hence one's action + payoff.

Brain vs. skull. (Both are needed).

I) Current Expectations on future events.

II Current expectations of future *expectations* on the same future events.

We may *expect* our current expectation to *change*, to become both more *definite* (precise, unambiguous) and more *reliable* (valid, accurate). Considerable Type I Uncertainty is a prerequisite for this (necessary) but does not *imply* it (not sufficient). Marschak-Nelson try to explain ~~Type I~~ flexibility, *including* Type II, in terms of Type I uncertainty (described by prob dist).

[illegible] is models of the theorems (many axioms or beliefs) [illegible] will allow him to "finds of "counterexamples" to decision rules, instead of stumbles isolated "counterexamples."

Explain motivation for a) rules, and (b) for axioms. When will you want to improve ?

Distinguish (Koopman) between [illegible] preferences and intuitive tests of consistency of judgment; the latter models, too, must be judged by their consequences (aesthetic [illegible]ations only part of the payoffs; Samuelson).

Note how "subjectivist" theory treats "objective" probs, where appropriate. (Neutrality, etc.; intuitive [illegible] about lotteries involving "objective" probs).

Axioms vs. "coherence."

Formulate axioms of choice ⟹ Koopman/Smith behavior.

Implications of ordinal comparisons between p_i and p_j, α_i and α_j.

When will Sure-thing Principle Apply?

Possibility that the simplifications and arbitrary elements in a model or rule correspond precisely (Schelling) to simplifications "we" make in practice, to ease calculation, speed decision.